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On Collinearization of Quarks in Quark-Gluon Decays of Heavy Orthoquarkonia

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ABSTRACT

The decay of heavy orthoquarkonium into quark-antiquark pair and two gluons is considered. The differential probability of the decay in the tree approximation is calculated and the final quark mass influence on the quark and gluon distribution functions is studied. It is shown that the collinear strengthening of the bottomonium decay probability takes place at the $u\bar{u}$-, $d\bar{d}$- and $s\bar{s}$-pair production and is absent at the $c\bar{c}$- production.

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The decays of heavy quarkonia give the useful information on the dynamics of quarks and gluons and on the processes of jet production of hadrons. In particular the many-particle decays such as ${}^1S_0 \to 3g,\ q\bar{q}g$ and ${}^3S_1 \to 4g,\ q\bar{q}gg$ are of great interest as the processes giving the immediate information on $q\bar{q}g$- and $3g$- interactions. The nonabelian nature of ggg-interaction manifest itself, for example, in the distribution on invariant masses of two particles,[1,2] as the acomplanarity of four particle decays,[3] as the collinearization of the gluons[4] and as some other effects. It is worth noting that the four particle decays of heavy orthoquarkonia are more available for experimental investigations because of rather great probability of direct production of orthoquarkonia in e^+e^-- and $p\bar{p}$– collisions. By this reason the detail theoretical analysis of these decays is interesting on purpose to find the optimal conditions for their experimental observation.

In this work we consider the decay of heavy orthoquarkonium with spin-parity $J^{PC} = 1^{--}$ into quark-antiquark pair and two gluons. The amplitude and differential probability of this process in tree approximation and some angle and energy distributions of quarks and gluons are obtained and discussed.

The process ${}^3S_1(Q\bar{Q}) \to q\bar{q}gg$ is described in tree approximation by six diagrams shown on Fig. 1. The amplitude of this process in the initial static quarks limit in the rest frame of quarkonium can be presented in the form:

$$M_{ab}^{\alpha\beta}({}^3S_1(Q\bar{Q}) \to q\bar{q}gg) = \frac{d_{abc}(t_c)_{\alpha\beta}}{4\sqrt{N_c}} \frac{g_{st}^4 \psi(0)}{4\sqrt{\epsilon_1\epsilon_2\omega_1\omega_2}} \frac{(w_{\bar{Q}}^T \sigma_2 \vec{\sigma} w_Q)}{4m^2p^2} \vec{V}, \tag{1}$$

where d_{abc} $(a, b, c = 1, 2, \ldots, N_c^2 - 1)$ are symmetrical constants of $SU(N_c)$, t_c are generators of $SU(N_c)$ group normalized as $Sp(t_a t_b) = \delta_{ab}/2$, $\psi(0)$ is the nonrelativistic wave function of quarkonium in the coordinate space, w_Q and $w_{\bar{Q}}$ are two-component spinors of initial quark and antiquark, $p = p_1 + p_2$, p_1 and p_2 are 4-momenta of final quark and antiquark, m is a mass of the initial quark, g_{st} is a color charge, $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are Pauli matrices.

The vector function $\vec{V}$ in (1) depends on momenta and polarizations of final quarks and gluons and in the three-transversal gauge takes the rather simple form:

Fig. 1. Diagrams of the decay ${}^3S_1(Q\bar{Q}) \to q\bar{q}gg$ in the tree approximation.

$$
\begin{aligned}
\vec{V} &= (\vec{e}_1^{\,+}\vec{e}_2^{\,+})\vec{j}^{\,-} + (\vec{e}_1^{\,-}\vec{e}_2^{\,-})\vec{j}^{\,+} + (\vec{e}_1^{\,+}\vec{j}^{\,+})\vec{e}_2^{\,-} + (\vec{e}_1^{\,-}\vec{j}^{\,-})\vec{e}_2^{\,+} \\
&+ (\vec{e}_2^{\,+}\vec{j}^{\,+})\vec{e}_1^{\,-} + (\vec{e}_2^{\,-}\vec{j}^{\,-})\vec{e}_1^{\,+} + 4\Big\{\Big[1 - \frac{2mp_0}{(kp)}\Big]\vec{j} + \frac{2m\vec{p}}{(kp)}j_0\Big\} \\
&+ ij_0\Big[1 - \frac{4m^2}{(kp)}\Big]\Big\{[\vec{e}_1^{\,+}, \vec{e}_2^{\,-}] - [\vec{e}_2^{\,-}, \vec{e}_1^{\,+}]\Big\},
\end{aligned} \tag{2}
$$

where $\vec{e}_i^{\,\pm} = \vec{e}_i \pm i[\vec{n}_i, \vec{e}_i]$, $\vec{n}_i = \vec{k}_i/\omega_i$, $e_i = \{0, \vec{e}_i\}$, $k_i = \{\omega_i, \vec{k}_i\}$ are 4-vectors of polarization and momentum of i-th gluon ($i = 1, 2$), $(\vec{n}_i\vec{e}_i) = 0$, $\vec{j}^{\,\pm} = \vec{j} \pm 2mi[\vec{j}, \vec{p}]/(kp)$, $k = k_1 + k_2$, $j = \{j_0, \vec{j}\} = \bar{u}(p_1)\gamma u(-p_2)$ is a four current of final $q\bar{q}$-pair, $u(p_1)$ and $u(-p_2)$ are Dirac spinors of final quark and antiquark, $\gamma = \{\gamma_0, \vec{\gamma}\}$ are Dirac matrices.

The corresponding to (1) and (2) differential probability averaged over three spin states of initial orthoquarkonium and summed over polarizations and colors of final particles may be presented in the form:

$$
dW = \frac{F_c}{6\pi^4}\frac{\alpha_s^4 \mid \psi(0) \mid^2 Q_{\mu\nu}G_{\mu\nu}}{[p^2(kp)(Pk_1)(Pk_2)]^2}\delta(P - p - k)\frac{d\vec{p_1}d\vec{p_2}d\vec{k_1}d\vec{k_2}}{\epsilon_1\epsilon_2\omega_1\omega_2}, \tag{3}
$$

$$
F_c = \frac{(N_c^2 - 4)(N_c^2 - 1)}{32N_c^2},
$$

where F_c is the color factor of $SU(N_c)$- group, for $SU(3)$ $F_c = 5/36$, $P = \{2m, \vec{0}\}$ is the four momentum of quarkonium in the rest frame, $\alpha_s = g_{st}^2/(4\pi)$ is the strong coupling constant, $Q_{\mu\nu}$, $G_{\mu\nu}$ are symmetrical tensor functions of momenta of quarks and gluons correspondingly. The expressions for $Q_{\mu\nu}$ and $G_{\mu\nu}$ obtained by us have the structure such as that of the tensors $L_{\mu\nu}$, $H_{\mu\nu}$ of Ref. 5.

Integrating (3) over quark and antiquark momenta we obtain the probability distribution in the energies $x_i = \omega_i/m$ of the gluons and angle ϑ_g between their momenta in the form:

$$
\frac{dW}{dx_1dx_2d\cos\vartheta_g} = F_c\frac{\alpha_s^4 \mid \psi(0) \mid^2}{36\pi m^2}\frac{F_g}{\eta_g\xi_g^2}\Big(1 + \frac{2\mu^2}{\eta_g}\Big)\sqrt{1 - \frac{4\mu^2}{\eta_g}}, \tag{4}
$$

$$
\begin{aligned}
F_g &= 8x_1x_2[12(1 + \cos^2\vartheta_g) - 8(x_1 + x_2)(1 - \cos\vartheta_g + \cos^2\vartheta_g) \\
&+ 4(1 - \cos\vartheta_g)[2(x_1 + x_2)^2 - x_1x_2(1 - \cos\vartheta_g - \cos^2\vartheta_g)] \\
&- 8x_1x_2(x_1 + x_2)(1 - \cos^2\vartheta_g) + x_1^2x_2^2(1 - \cos\vartheta_g)^3(3 - \cos\vartheta_g)], \\
\eta_g &= (P - k)^2/m^2 = 4(1 - x_1 - x_2) + 2x_1x_2(1 - \cos\vartheta_g), \\
\xi_g &= ((Pk) - k^2)/m^2 = 2(x_1 + x_2) - 2x_1x_2(1 - \cos\vartheta_g),
\end{aligned}
$$

Fig. 2. Angle distributions of gluons at $x_1 = x_2 = x$ and quarks at $y_1 = y_2 = y$ in the decay ${}^3S_1(b\bar{b}) \to s\bar{s}gg$: a) $x = 0.4$ or $y = 0.4$, b) $x = 0.6$ or $y = 0.6$, c) $x = 0.8$ or $y = 0.8$.

Fig. 3. Angle distributions of gluons at $x_1 = x_2 = x$ and quarks at $y_1 = y_2 = y$ in the decay ${}^3S_1(b\bar{b}) \to c\bar{c}gg$: a) $x = 0.4$ or $y = 0.4$, b) $x = 0.6$ or $y = 0.6$, c) $y = 0.8$.

where $\mu = m_q/m$ is mass ratio of final and initial quarks.

Similarly integrating (3) over momenta of gluons we obtain the probability distribution in the energies $y_i = \epsilon_i/m$ of quarks and angle ϑ_q between their momenta in the form:

$$\frac{dW}{dy_1 dy_2 d\cos\vartheta_q} = F_c \frac{\alpha_s^4 |\psi(0)|^2}{36\pi m^2} \frac{F_q}{\eta_q^2 \xi_q^2 (2 - y_1 - y_2) v^4}, \tag{5}$$

$$\begin{aligned}
\eta_q &= p^2/m^2 = 2\Big[\mu^2 + y_1 y_2 - \cos\vartheta_g \sqrt{(y_1^2 - \mu^2)(y_2^2 - \mu^2)}\Big], \\
\xi_q &= ((Pp) - p^2)/m^2 = 2(y_1 + y_2) - \eta_q, \\
v &= \sqrt{1 - \frac{4m^2 k^2}{(Pk)^2}} = \frac{\sqrt{(y_1 + y_2)^2 - \eta_q}}{2 - y_1 - y_2},
\end{aligned}$$

where v is the center-of-mass velocity of the gluon pair in the rest frame of quarkonium,

$$F_q = P_1(y_1, y_2, v^2) + \frac{1 - v^2}{v} \ln\Big|\frac{1 + v}{1 - v}\Big| P_2(y_1, y_2, v^2), \tag{6}$$

P_1 and P_2 are some polynomials of degree six in v^2, which have a rather complicated form and we don't adduce them here.

It's convenient further to use the distributions (4) and (5) normalized as

$$
\begin{aligned}
f_g &= \frac{1}{\alpha_s W_{3g}} \frac{dW}{dx_1 dx_2 d\cos\vartheta_g}, \\
f_q &= \frac{1}{\alpha_s W_{3g}} \frac{dW}{dy_1 dy_2 d\cos\vartheta_q}, \\
W_{3g} &= 2F_c \frac{16(\pi^2-9)}{9} \frac{\alpha_s^3 \mid \psi(0) \mid^2}{m^2},
\end{aligned}
\tag{7}
$$

where W_{3g} is the three-gluonic decay probability of orthoquarkonium. The gluon (quark) distributions f_g (f_q) as the functions of angle ϑ_g (ϑ_q) between the momenta of the gluons (quarks) at equal energies $x_1 = x_2 = x$ ($y_1 = y_2 = y$) are presented on Fig. 2 and 3 for bottomonium decays ${}^3S_1(b\bar{b}) \to s\bar{s}gg$ and ${}^3S_1(b\bar{b}) \to c\bar{c}gg$ correspondingly. These distributions exhibit two interesting peculiarities.

First, the analysis of the quark distribution f_q shows that the probability of the production of light $q\bar{q}$- pair and two gluons can significantly increase as the angle between quarks decreases. This collinear effect has a simple nature – the decrease of the denominator in the propagator of the virtual gluon as the angle between light quarks decreases. As shown in Fig. 2, 3 this effect exhibits itself in the bottomonium decay with production of $s\bar{s}$- pair (it's more exhibited in the decays with production of $u\bar{u}$-, $d\bar{d}$- pairs) but it's absent in the decay with production of heavy $c\bar{c}$- pair. It means, that in the decays ${}^3S_1(b\bar{b}) \to q\bar{q}gg$ of bottomonium the production of light $u\bar{u}$-, $d\bar{d}$- and $s\bar{s}$- pairs at small angle between quark and antiquark is more probable than that of $c\bar{c}$- pair. The corresponding 4-jet events can be possibly available for experimental observation by appropriate angle resolution.

Secondly, the production of hard $q\bar{q}$- pair with large angle between q- and $\bar{q}$- quarks accompanied by production of two soft gluons is more probable too (see quark curve c) on Fig. 2, 3).

The angle distributions of the quarks and gluons obtained and discussed by us may be useful for the experimental searches and investigations of the four-jet events from the decays of heavy quarkonia.

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References

1. K. Koller, K.H. Streng, T.F. Walsh and P.M. Zerwas P.M., *Nucl. Phys.* **B206** (1982) 273.
2. K.H. Streng *Z. Phys.* **C27** (1985) 107.
3. T. Muta and T. Niuya *Progr. Theor. Phys.* **68** (1982) 1735.
4. A.D. Smirnov *Yad. Fiz.* **47** (1988) 1380.

5. L. Clavelli, P.H. Cox and B. Harms *Phys. Rev.* **D31** (1985) 78.